

# Mass generation with adjoint fermions

Yu.A.Simonov

State Research Center

Institute of Theoretical and Experimental Physics,  
Moscow, 117218 Russia

February 24, 2013

## Abstract

The QCD-like gauge theory with adjoint fermions is considered in the field correlator formalism and the total spectrum of mesons and glueballs is obtained in agreement with available lattice data. A new state of a white fermion appears, as a bound state of the adjoint fermion and gluon with the mass close to that of glueball. It is shown, that the main features of spectra and thermodynamics of adjoint fermions can be explained by this new bound state.

## 1 Introduction

Gauge theories with adjoint quarks are an object of intensive studies both on the lattice [1]-[6] and in theory [7, 8], see [9] for a review of theoretical models. This interest is mostly connected with the technicolor models, which suggest adjoint techniquarks as the source of quark masses and interaction at high scale [10, 11].

At the same time theories with adjoint quarks present an interesting example of theories, where one can test mechanisms developed in the framework of the Field Correlator Method (FCM)[12] for confinement [13], chiral symmetry breaking (CSB) [14], and temperature phase transition [15], known for fundamental fermions .

In particular, lattice data with adjoint fermions [4, 5, 6] exhibit a completely different hadron spectra, where an equivalent of the pion is heavier,

than the glueball. Moreover, CSB seems to have two different thresholds in temperature [1]: one, coinciding with deconfinement temperature  $T_{\text{dec}}$ , and another much higher,  $T_\chi \simeq 6.6 T_{\text{dec}}$ , where the remnants of CSB disappear. We shall argue below, that these features can be explained in the framework of FCM in a simple way. The basic element, as will be shown below is, that an adjoint fermion can be bound with a gluon, forming a colorless fermion with nonzero mass, which we call gluequark. Moreover, also the adjoint fermion itself can acquire mass, coupling to gluequark, and this creates a completely different picture of CSB – and of fermion mass generation in principle. These both facts can be important, since this new mechanism is different from the original mass-generating mechanism of ordinary quarks, suggested in technicolor theories.

It is a purpose of the present paper to apply to the gauge theory with adjoint fermions, (which we call shortly AdQCD) the Field Correlator Method, which has provided a selfconsistent mechanism of confinement [13], and explained the interconnection of chiral symmetry restoration [14] and temperature deconfinement [15] in case of fundamental quarks (see [12] for reviews). Following this method we define an exact path-integral representation (the so-called Fock-Feynman-Schwinger representation (FFSR)) [16] for fermions and gluons in the confining background field, which is characterized by gauge invariant correlators, and derive relativistic Hamiltonians for white systems of quarks and gluons ( $qg$ ), two gluons ( $gg$ ), and mesons ( $q\bar{q}$ ) and baryons ( $qqq$ ).

This enables us to obtain the corresponding spectra of masses. In doing so we encounter in the interaction potential  $V(R) = V_{\text{conf}}(R) + V_1(R)$  an interesting phenomenon, which is not known for fundamental fermions in the quenched approximation: the static potential  $V_{\text{conf}}$  does not grow beyond some distance  $R_{cr}$  which corresponds to the threshold for creation of two white fermions – bound state  $qg$ . This is similar to the situation in the unquenched QCD, where the growth of static potential stops beyond some distance, but in AdQCD this transition is more sharp, as shown by lattice data [1, 17]. Another unexpected feature – the strong interaction  $V_1(R)$  in the deconfined phase, which is able to bind quark and gluon into a white fermion, as it is done by  $V_{\text{conf}}(R)$  in the confining phase.

In the last part of the paper we discuss thermal properties of AdQCD and define  $T_{\text{deconf}}$  and show, that it coincides with the first CSB transition, which we call  $T_{\chi 1}$ ,  $T_{\chi 1} = T_{\text{deconf}}$ . However, we show, that the remnants of mass of the white ( $qg$ ) state can be nonzero beyond  $T_{\chi 1}$  and this part of mass

(due to the nonconfining correlator  $D_1$ ) produces chiral condensate, which is gradually decreasing with temperature, finally disappearing at  $T_{\chi 2}$ , which signals a full recovery of chiral symmetry.

Our results may be of importance for technicolor theories (TC), since they provide a new mechanism of fermion mass generation, and moreover, they allow to calculate these masses explicitly. The paper is organized as follows: the section 2 is devoted to the derivation of the AdQCD Hamiltonian from FFSR, for all possible white combinations. In section 3 spectra of all states are calculated and compared with lattice data. At the end of the section the chiral properties of mesons are discussed and compared to lattice data. In section 4 the temperature phase transition is discussed for confinement and CSB phenomena. Section 5 contains summary and perspectives.

## 2 Gauge fields with adjoint quarks. Green's functions

We start with the action for AdQCD in the Euclidean space-time

$$S = \frac{1}{4} \int (F_{\mu\nu}^a(x))^2 d^4x - i \int \psi^+ (\hat{D} + m) \psi d^4x, \quad (1)$$

where  $\psi$  is an adjoint fermion, which can be written both in double fundamental and adjoint indices

$$\psi_{\alpha\beta} = \psi^a t_{\alpha\beta}^a, \quad \psi^a(x) \rightarrow U_{ab}^+(x) \psi^b(x), \quad \psi_{\alpha\beta}(x) \rightarrow U_{\alpha\alpha'}^+ \psi_{\alpha'\beta'} U_{\beta'\beta} \quad (2)$$

$$\psi_a^+ (D_\mu)_{ab} \psi_b = \psi_a^+ (\partial_\mu \delta_{ab} + g A_\mu^c f^{abc}) \psi_b. \quad (3)$$

The term  $\psi^+ \hat{D} \psi$  can be also be written in fundamental indices, where gauge invariance is evident, as

$$\psi_{\alpha\beta}^+ \hat{D}_{\beta\gamma} \psi_{\gamma\alpha} \rightarrow U^+ \psi^+ U U^+ \hat{D} U U^+ \psi U = \psi^+ \hat{D} \psi.$$

One can now form white combinations of  $q, g$  as  $\langle \text{in} |$  and  $|\text{out} \rangle$  hadron states in AdQCD. One can do it both with the total vector field  $A_\mu$ , and (more conveniently) in the background field formalism [18], separating vacuum background field  $B_\mu$ ,  $A_\mu = B_\mu + a_\mu$ , where gauge transformations are

$$B_\mu \rightarrow U^+ (B_\mu + \frac{i}{g} \partial_\mu) U, \quad a_\mu \rightarrow U^+ a_\mu U. \quad (4)$$

In what follows we shall work mostly with the field  $a_\mu$ , as it was done in [19, 20] for glueballs and gluelumps respectively, indicating also the corresponding white combinations with  $A_\mu$  (or  $F_{\mu\nu} = (\mathbf{E}, \mathbf{H})$ ).

Fermions ( $\psi_{\alpha\beta}$  or  $\psi^a$ ) and gluons ( $(A_\mu)_{\alpha\beta}$  or  $A_\mu^a$ ) can be depicted by thin straight lines and by wavy lines for gluons. In this way one can define mesons,

$$\Psi_i = \psi_a^+ \Gamma_i \psi_a = \psi_{\alpha\beta}^+ \Gamma_i \psi_{\beta\alpha}, \quad (5)$$

$gg$  glueballs, and  $ggg$  glueballs.

$$\Phi_i^{(2)} = \text{tr}(a_\mu T_{\mu\nu}^{(i)} a_\nu), \quad \Phi_i^{(3)} = \text{tr}(T_{\mu\nu\lambda}^{(i)} a_\mu a_\nu a_\lambda), \quad (6)$$

where  $T_{\mu\nu}^{(i)}$ ,  $T_{\mu\nu\lambda}^{(i)}$  contain covariant derivatives, in simplest cases for  $0^{++}gg$  state  $T_{ik}^{(1)} = \delta_{ik}$ ,  $i, k = 1, 2, 3$ .

The equivalent form for baryons is  $B_i = f^{abc} \psi^a \psi^b \psi^c$ . There is however in AdQCD a gauge invariant object, which is missing in the standard QCD, namely the  $qg$  bound state, which we call gluequark,

$$Q_\mu(x) = \text{tr}(\psi(x) a_\mu(x)), \quad Q_\mu^+(x) = \text{tr}(\psi^+(x) a_\mu(x)), \quad (7)$$

and more complicated versions with a matrix operator  $\Gamma$  between  $a_\mu$  and  $\psi$ . The equivalent form can be written with  $F_{\mu\nu}$ ,

$$Q_i^E = \text{tr}(\psi(x) E_i(x)), \quad Q_i^H = \text{tr}(\psi(x) H_i(x)). \quad (8)$$

We are now in position to write Green's functions for  $q, g$  and finally for mesons  $\Psi_i$ , glueballs  $\Phi_i$  and gluequark  $Q_\mu$ .

The first two FFSR for  $q, g$  have been written before [16], and one should only account for the adjoint representation of the fermion

$$\begin{aligned} G_q(x, y) &= \langle \psi(x) \psi^+(y) \rangle_q = \langle x | (m + \hat{D})^{-1} | y \rangle = \\ &= \langle x | (m - \hat{D})(m^2 - \hat{D}^2)^{-1} | y \rangle = (m - \hat{D}) \int_0^\infty ds (Dz)_{xy} e^{-K_q} \phi_q(x, y) \end{aligned} \quad (9)$$

where  $(Dz)_{xy}$  is the path integral element,  $K_q = m_q^2 s + \frac{1}{4} \int_0^s \left( \frac{dz_\mu}{d\tau} \right)^2 d\tau$ , and

$$\phi_q(x, y) = P_A \exp(ig \int_y^x A_\mu(z) dz_\mu) P_F \exp(g \int_0^s d\tau \sigma_{\mu\nu} F_{\mu\nu}), \quad (10)$$

where  $P_A, P_F$  are ordering operators and  $\sigma_{\mu\nu} F_{\mu\nu} = \begin{pmatrix} \boldsymbol{\sigma} \mathbf{H} & \boldsymbol{\sigma} \mathbf{E} \\ \boldsymbol{\sigma} \mathbf{E} & \boldsymbol{\sigma} \mathbf{H} \end{pmatrix}$ .

In a similar way one can write the gluon Green's function of the field  $a_\mu$  in the background field  $B_\mu$  [21]

$$G_g(x, y)_{\mu\nu} = \langle x | (D_\lambda^2 \delta_{\mu\nu} - 2igF_{\mu\nu})^{-1} | y \rangle = \int_0^\infty ds (Dz)_{xy} e^{-K_g} \phi_{\mu\nu}(x, y), \quad (11)$$

where  $K_g = \frac{1}{4} \int_0^s \left( \frac{dz_\mu}{d\tau} \right)^2 d\tau$  and  $\phi_{\mu\nu}$  is

$$\phi_{\mu\nu}(x, y) = P_A \exp(ig \int_y^x B_\lambda dz_\lambda) P_F \exp(2g \int_0^s d\tau F_{\sigma\rho}(z(\tau))_{\mu\nu}) \quad (12)$$

Now the Green's function of any white two-component system ( $q\bar{q}$ ,  $gg$  or  $qg$ ) averaged over background field  $B_\mu$  can be written in the form

$$G_{ik}(x, y) = S \int_0^\infty ds \int_0^\infty ds' (Dz)_{xy} (Dz')_{xy} e^{-K_i - K_k} \langle W_F \rangle \quad (13)$$

where  $i, k = q\bar{q}, gg, qg, \bar{q}g$ ;  $S$  contains possible  $(m - \hat{D})$  operators and  $\langle W_F \rangle$  is the Wilson loop with insertions of operators  $F_{\mu\nu}$ ,

$$\langle W_F \rangle = \text{tr} P_B P_F \langle \exp \{ ig \int_C B_\mu dz_\mu + \sum_i 2g \int (s^{(i)} F) d\tau \} \rangle_B. \quad (14)$$

Here  $(s^{(q)} F) = \frac{1}{2} \sigma_{\mu\nu} F_{\mu\nu}$  for a fermion (of spin  $\frac{1}{2}$ ) and  $(s^g F) = (\mathbf{s}^g \mathbf{H} + \tilde{\mathbf{s}}^g \mathbf{E})_{\mu\nu} = -iF_{\mu\nu}$ , and gluon spin operators are introduced as follows [19]

$$(s_m^{(g)})_{ik} = -ie_{mik}, \quad (\tilde{s}_m^g)_{i4} = -i\delta_{im}. \quad (15)$$

The relativistic Hamiltonian can now be derived from (13) as in [22], assuming smooth trajectories for both partners in  $G_{ik}$  (which actually implies that corrections from  $Z$  graphs with backward-in-time motion are included in the selfenergy terms, explicitly written). In this way one obtains (see [22], [23] for details of derivation)

$$ds(D^4 z)_{xy} \rightarrow (D^3 z)_{\mathbf{x}\mathbf{y}} \frac{D\omega}{2\bar{\omega}}, \quad (16)$$

and  $\bar{\omega}$  is the average quark (or gluon) energy inside hadron (the bar sign over  $\omega$  will be omitted in what follows). In this way, following [22], [23] one obtains both hadron coupling constant  $f_h$ , and the relativistic Hamiltonian in the c.m. system, which we shall write for simplicity for the zero angular

momentum as a common form in three cases: a) for mesons; b) for gluballs; c) for gluequark

$$H = \sum_i \sqrt{\mathbf{p}^2 + m_i^2} + \sigma_{\text{adj}} r + V_c^{(\text{adj})}(r) + V_{ss} \quad (17)$$

and its einbein form [22]

$$H_\omega = \sum_i \left( \frac{\omega_i}{2} + \frac{\mathbf{p}^2 + m_i^2}{2\omega_i} \right) + \sigma_{\text{adj}} r + V_c^{(\text{adj})}(r) + V_{ss} \quad (18)$$

where  $V_c^{(\text{adj})}(r)$  is an effective gluon exchange potential, for gluon  $m_i = 0$  and  $V_{ss}$  is the spin-dependent interaction, and we shall be interested below in  $L = 0$  states and keep only the hyperfine interaction  $V_{ss}$ , which is slightly modified for adjoint sources [19] (gluon exchanges in  $V_c^{(\text{adj})}$  are strongly reduced by BFKL loop corrections, see [19] for details)

$$V_{ss} = \frac{\mathbf{s}^{(i)} \mathbf{s}^{(j)}}{3\omega_i \omega_j} V_4(r), \quad V_4(r) = 5\pi C_2(\text{adj}) \alpha_s \delta^{(3)}(r) \quad (19)$$

Here  $\omega_i$  in the einbein approximation [22] (better than 5% for lowest states) is found from the condition on the resulting mass  $M$ ,  $H_\omega \Psi = M \Psi$ ,

$$\frac{\partial M(\omega_1, \omega_2)}{\partial \omega_i} = 0, \quad i = 1, 2. \quad (20)$$

The mass  $M$  – the eigenvalue of the Hamiltonian (18) – can be written as

$$M_n = \sum_{i=1}^i \frac{1}{2} \left( \frac{m_i^2}{\omega_i} + \omega_i \right) + \varepsilon_n(\tilde{\omega}) + \Delta M_{ss}, \quad \tilde{\omega} = \frac{\omega_1 \omega_2}{\omega_1 + \omega_2}, \quad (21)$$

$$\Delta M_{ss} = \frac{5\alpha_s \sigma_{\text{adj}}}{2(\omega_1 + \omega_2)} (\mathbf{s}_i \mathbf{s}_j)$$

where  $\varepsilon_n$  is expressed via dimensionless numbers  $a_n$

$$\varepsilon_n = (2\tilde{\omega})^{-1/3} (\sigma_{\text{adj}})^{2/3} a_n. \quad (22)$$

We shall be interested in the lowest states  $a_0 = 2.338$ ,  $a_1 = 4.088$  for  $L = 0$  and  $n_r = 0$  and 1 respectively.

Eq. (20) with the help of Eqs. (21), (22) yields  $\omega_1, \omega_2$

$$\omega_i^2 = m_i^2 + \frac{(\sigma_{\text{adj}})^{2/3} a_n (2\tilde{\omega})^{2/3}}{3}, \quad i = 1, 2. \quad (23)$$

For  $m_1 = m_2 = 0$  one obtains  $\omega = \left(\frac{a_n}{3}\right)^{3/4} \sqrt{\sigma_{\text{adj}}}$ , while for heavy quarks,  $m_i > \sqrt{\sigma_{\text{adj}}}$ , one has

$$\omega_i^2 \approx m_i^2 + \frac{(\sigma_{\text{adj}})^{2/3} a_n \tilde{m}^{2/3}}{3} + \dots \quad (24)$$

Finally, the hyperfine correction  $\Delta M_{ss}$  is

$$\Delta M_{ss} = \frac{5N_c}{6} \frac{\alpha_s(hf) \sigma_{\text{adj}}}{(\omega_1 + \omega_2)} \mathbf{s}_1 \mathbf{s}_2, \quad (25)$$

$$\mathbf{s}_1 \mathbf{s}_2 = \frac{J(J+1) - s_1(s_1+1) - s_2(s_2+1)}{2},$$

where  $J, s_1, s_2$  are spins of the bound system and of its components respectively,  $\alpha_s(hf)$  is the effective  $\alpha_s$  in the hyperfine interaction, we take it  $\alpha_s(hf) = 0.25$ , and we neglect the nonperturbative part of hyperfine interaction, see [19] for a discussion.

One should add a short discussion on  $V_c^{(\text{adj})}(r)$ .

Naively one could assume, that gluon exchange potential  $V_c(r)$  between adjoint quarks or gluons is the same, as between fundamental quark and antiquark, but multiplied by the Casimir factor  $9/4$ . However, one can argue, as it was done in [19], that such strong interaction is largely reduced due to formation of the quark-gluon chains of BFKL type, which produces a rather weak resulting interaction, which can be deduced from the relatively small shift  $\Delta = \alpha_P(0) - 1$  of the intercept. This conclusion is also confirmed by lattice calculations in [24], where the resulting masses for glueballs do not show significant contribution from  $V_c(r)$ . Therefore we neglect  $V_c(r)$ ,  $\Delta M_c$  in the first approximation and keep perturbative gluon contribution only in the spin-dependent interaction  $V_{ss}$ .

Looking at the Hamiltonian (18) in the case of zero fermion current mass  $m_i$ , one can see, that the resulting mass eigenvalues in the systems  $gg$  and  $q\bar{q}$  may differ for the zero quark mass only due to hyperfine interaction, which is large in both systems due to adjoint Casimir factor  $9/4$ . E.g. in lattice calculations the mass difference between  $2^{++}$  and  $0^{++}$  glueballs is around 0.7 GeV.

We are now in position to calculate the glueball and gluequark bound states, using the Hamiltonian (18), (19) with the condition (20). The result for glueballs are essentially the same as in [19], here we only slightly change  $\alpha_s(hf)$  in the  $hf$  interaction (19), taking in this paper  $\alpha_s^{(eff)} = 0.25$ . Then the difference between  $qg$  and  $gg$  bound states is only in the  $ss$  term due to different spin values: one obtains total spin value  $J = 0, 2$  for  $gg$  and  $J = \frac{1}{2}, \frac{3}{2}$  for  $qg$  states.

Results are given in the Table 1 for  $\sigma_f = 0.18 \text{ GeV}^2$  and assuming zero masses of adjoint quarks.

Table 1: The  $L = 0$  masses of  $gg$  and  $qg$  bound states,  $\frac{m_{qg}}{\sqrt{\sigma_f}}, \frac{m_{gg}}{\sqrt{\sigma_f}}$  for  $N_c = 3$ ,  $\alpha_s(hf) = 0.25$  in comparison with lattice data [24].

System	$gg$		$qg$	
$J^P$	$0^+$	$2^+$	$\frac{1}{2}^-$	$\frac{3}{2}^-$
$m/\sqrt{\sigma_f}$ , this work	3.56	5.30	4.15	5.02
$m/\sqrt{\sigma_f}$ , $SU(3)$ [24]	3.55	4.78	-	-
$m/\sqrt{\sigma_f}$ , $SU(2)$ [24]	3.78	5.45	-	-

The r.m.s. radii of these bound states are around  $R_0 \approx (0.3 \div 0.4) \text{ fm}$  and hence one can expect, that for  $R \gtrsim R_{cr} \approx (0.6 \div 0.8) \text{ fm}$  the total potential between adjoint quarks does not grow and becomes flat. Qualitatively this picture agrees with what was found on the lattice in the  $SU(3)$  theory with adjoint fermions [1], [17].

Of special interest are vector  $V$  and pseudoscalar PS meson masses, which can be computed from the current quark masses (assumed here to be vanishingly small), or for the quark masses equal to  $m(qg)$ . The resulting masses for  $m_q = 0$  are expressed in terms of  $\sigma$  and were computed in [22]-[25]. (Note, that the selfenergy correction [26] is important here). Results for  $L = 0$ ,  $n = 0$  with the assumption of zero quark mass are easily found from (21),(25) (here we do not exploit the additional Nambu-Goldstone mechanism of PS mass suppression, as was done in [14, 25], which would drive  $m_{PS}$  to zero for zero quark mass). One obtains for adjoint quark-antiquark states

$$m_{PS}(m_q = 0) = 0.685 \text{ GeV}, \quad m_V = 1.075 \text{ GeV}. \quad (26)$$

These values are in strong disagreement with lattice data for  $n_f = 2$  in



[2, 4], where

$$\frac{M_{PS}}{\sqrt{\sigma_{\text{adj}}}} \approx 7, \quad \frac{M_V}{M_{PS}} \approx 1.04. \quad (27)$$

Therefore we shall explore now another dynamics, where the bare quark mass coincides with  $m_{qg} \left(\frac{1}{2}\right)$ . As one can see in Table 1,  $m_{qg} \left(\frac{1}{2}\right) \cong 2.8\sqrt{\sigma_{\text{adj}}}$ , and the average energy of the effective  $(q\bar{q})$  can be obtained from (23, (24), which reduces to the equation for  $x = \left(\frac{\sqrt{\sigma_{\text{adj}}}}{\omega}\right)^{2/3}$ :  $1 = \frac{m_{qg}^2}{\sigma}x^3 + \frac{a_0}{3}x^2$ . Here  $\Delta M_{ss} = \Delta_{ss}\sqrt{\sigma_{\text{adj}}} \mathbf{s}_1\mathbf{s}_2$ ,

$$M_{PS}(M_q = m_{qg}) \cong 2m_{qg} + \frac{a_0\sigma_{\text{adj}}^{2/3}}{m_{qg}^{1/3}} - \Delta_{ss}\frac{3}{4} = (7.25 - \frac{3}{4}\Delta_{ss})\sqrt{\sigma_{\text{adj}}}, \quad (28)$$

$$M_V(M_q = m_{qg}) = 2m_{qg} + \frac{a_0\sigma_{\text{adj}}^{2/3}}{m_{qg}^{1/3}} + \Delta_{ss}\frac{1}{4} = (7.25 + \frac{1}{4}\Delta_{ss})\sqrt{\sigma_{\text{adj}}}. \quad (29)$$

One can assume, that the hyperfine interaction occurs at very small distances, when both quark and antiquark are bare, i.e. without clouds of gluons, which turn them into  $(qg)$  and  $(\bar{q}g)$  respectively, i.e. they have  $\omega_q^{(hf)} \approx \omega_g^{(hf)} = \left(\frac{a_0}{3}\right)^{3/4} \sqrt{\sigma_{\text{adj}}} = 0.83\sqrt{\sigma_{\text{adj}}}$ , and  $\Delta_{ss} = \frac{N_c}{3}0.393 \left(\frac{\alpha_s}{0.25}\right)$ .

Then  $\Delta_{ss} = \Delta_{gg} = 0.262$  for  $\alpha_s(hf) = 0.25$ ,  $N_c = 2$  and the masses are  $M_{PS} \cong 7.05\sqrt{\sigma_{\text{adj}}}$ ,  $M_V \cong 7.31\sqrt{\sigma_{\text{adj}}}$ , while the ratio is

$$\frac{M_V}{M_{PS}} = 1.037, \quad (30)$$

Table 2: Masses of  $PS$  and  $V$  states of  $q\bar{q}$ ,  $\frac{M_{PS}}{\sqrt{\sigma_{\text{adj}}}}$ ,  $\frac{M_V}{\sqrt{\sigma_{\text{adj}}}}$  and their ratio  $\frac{M_V}{M_{PS}}$  for  $\alpha_s = 0.25$  and  $N_c = 2$ .

	this work	[4,5]
$\frac{M_{PS}}{\sqrt{\sigma_{\text{adj}}}}$	7.05	$\approx 7$
$\frac{M_V}{\sqrt{\sigma_{\text{adj}}}}$	7.31	$\approx 7$
$\frac{M_V}{M_{PS}}$	1.037	1.04

The resulting values of  $M_{PS}, M_V$  are given in Table 2 in comparison with approximate values, deduced from [4, 5]. In a similar way one can consider mesonic states made from the gluequarks of spin  $\frac{3}{2}$ . In this case  $M_{ss} = 0.075\sqrt{\sigma_{\text{adj}}}\mathbf{s}_1\mathbf{s}_2$  and one has for the lowest and highest spin states

$$\frac{M_{PS}(J=0)}{\sqrt{\sigma_{\text{adj}}}} = 8.30 - \frac{15}{4} \cdot 0.075 = 8.02 \quad (31)$$

$$\frac{M(J=3)}{\sqrt{\sigma_{\text{adj}}}} = 8.30 + \frac{9}{4} \cdot 0.075 = 8.47 \quad (32)$$

with the ratio  $\frac{M(J=3)}{M_{PS}} = 1.056$ .

At this point one must consider the general picture of CSB and distinguish two possible mechanisms of CSB:

1. CSB due to confinement without fermion mass generation, valid for zero or small fermion mass generation,  $m < m_{\text{crit}}$ .
2. CSB due to large fermion mass,  $m > m_{\text{crit}}$ .

The difference between these two pictures lies in the mechanism of Nambu-Goldstone meson creation, which is valid in the case 1, and is absent in the case 2.

In our case the appearing fermion mass is the mass of  $qg$  state, which is large and therefore one can assume that no Nambu-Goldstone phenomenon can take place at our scale.

### 3 Thermodynamics of adjoint fermions

Thermodynamics of QCD with adjoint fermions was studied in [1],[2], where two phase transitions were found with  $T_{\text{chiral}}$  and  $T_{\text{deconf}}$ , and  $T_{\text{chiral}}/T_{\text{deconf}} \cong 6.65$  [1]. We shall discuss possible features of thermodynamics of AdQCD, using the formalism of FCM and we show that there are two chiral transitions, one at  $T_{\chi 1} = T_{\text{deconf}}$ , and another, at  $T_{\chi 2} = T_{\text{chiral}}$ .

In this framework the colorelectric field correlators are defined in a gauge invariant way as

$$\frac{g^2}{N_c} \text{tr}(E_i(x)\Phi(x,y)E_k(y)\Phi(y,x)) = \delta_{ik}(D^E(u) + D_1^E(u) + u_4^2 \frac{\partial D_1^E}{\partial u_4^2}) + u_i u_k \frac{\partial D_1^E}{\partial u^2}, \quad u \equiv x-y \quad (33)$$

give rise inside Wilson loop  $\langle W_F \rangle$  (14) to two interactions between  $q$  and  $\bar{q}$ ,  $q$  and  $g$ , or  $g$  and  $g$  at the temperature  $T$

$$V^E(r, T) = 2 \int_0^r (r - \lambda) \int_0^{1/T} d\nu (1 - \nu T) D^E(\sqrt{\lambda^2 + \nu^2}) \approx \sigma r (r \rightarrow \infty) \quad (34)$$

$$V_1^E(r, T) = \int_0^{1/T} d\nu (1 - \nu T) \int_0^r \lambda d\lambda D_1^E(\sqrt{\lambda^2 + \nu^2}) = \text{const} (r \rightarrow \infty) \quad (35)$$

In our previous discussion these potentials were referred to as  $V_{\text{conf}}$  and  $V_1$  respectively.

The physical picture of deconfinement, was suggested in [27] and further developed in [28],[15], (see[29] for a review), where agreement with lattice calculations is demonstrated.

It is based on the notion, that  $D^E$  (and hence  $\sigma$ ) vanishes at  $T \geq T_{\text{deconf}}$ . Moreover, this vanishing happens due to the fact, that the minimum of the total free energy  $F = -P$  (or maximum of the total pressure  $P$ ), consisting of vacuum energy density of gluonic fields and the free energy (pressure) of valent mesons, glueballs, or quarks and gluons, requires the confining part of vacuum energy density to vanish above  $T = T_{\text{deconf}}$ , as was found in lattice calculations [30].

One can write equality of  $P$  in two phases,  $P_I = P_{II}$ , where

$$P_I = |\varepsilon_{\text{vac}}^{\text{conf}}| + \chi_1(T), \quad P_{II} = |\varepsilon_{\text{vac}}^{\text{deconf}}| + P_{gl} + P_q \quad (36)$$

Here  $\chi_1(T)$ , is the hadronic gas pressure, which will be neglected with 10% accuracy. We also take quark mass equal to zero.

The gluonic energy density of the vacuum, expressed via gluonic condensate  $G_2$  is

$$\varepsilon_{\text{vac}} = \frac{\beta_0}{32} G_2, \quad G_2 = \frac{2\alpha_s \langle E_i^2 + H_i^2 \rangle_{\text{vac}}}{\pi} = \frac{3N_c}{\pi^2} (D^E(0) + D_1^E(0) + D^H(0) + D_1^H(0)) \quad (37)$$

Taking into account, that  $D_1^E(x)$  vanishes for  $x \rightarrow 0$ , and does not contribute to  $G_2$  [31] and the fact, that at  $T = 0$ ,  $D^E(0) = D^H(0)$  (and we keep this equality for  $T = T_{\text{deconf}}$ ) one obtains

$$|\varepsilon_{\text{vac}}^{\text{conf}}| \approx 2|\varepsilon_{\text{vac}}^{\text{deconf}}| \quad (38)$$

As a result, Eq. (36) yields for the transition temperature  $T_c$

$$T_c \equiv T_{\text{deconf}} = \left( \frac{|\beta_0| G_2}{64(p_g + p_q)} \right)^{1/4} \quad (39)$$

$$p_i = P_i/T^4, \quad i = g, f, a$$

Eq.(38) is applicable to  $SU(N_c)$  theories with  $n_f$  fundamental and  $n_a$  adjoint quarks, and

$$\beta_0 = \frac{11}{3}N_c - \frac{2}{3}n_f - \frac{4}{3}N_cn_a. \quad (40)$$

In the leading approximation of the Vacuum Dominance Approach [15] the pressure  $p_i$  is expressed via Polyakov lines and was calculated in [15]

$$p_g = \frac{2(N_c^2 - 1)}{\pi^2} L_a, \quad p_f = \frac{4N_cn_f}{\pi^2} L_f, \quad p_a = \frac{4(N_c^2 - 1)n_a}{\pi^2} L_a \quad (41)$$

where Polyakov lines  $L_a, L_f$  are expressed via  $V_1(r, T)$

$$L_f = \exp \left( -\frac{V_1(\infty, T)}{2T} \right), \quad L_a = \exp \left( -\frac{9V_1(\infty, \pi)}{8T} \right), \quad (42)$$

To find  $V_1(\infty, T)$  one can into account, that  $D_1(x)$  and hence  $V_1(r, T)$  are expressed via the gluelump with mass  $O(1 \text{ GeV})$  and were calculated in [31, 32] in comparison to lattice data [33], yielding approximately for fundamental quarks

$$V_1(\infty, T) = \frac{0.17 \text{ GeV}}{1.35 \left( \frac{T}{T_c} \right) - 1}; \quad V_1(\infty, T_c) = 0.5 \text{ GeV}. \quad (43)$$

Taking the same value of  $V_1(\infty, T_c) \approx 0.5 \text{ GeV}$  in the case of adjoint quarks, one obtains from (39) for  $N_c = 3$ ,  $n_a = 2$  and  $n_f = 0$  (as in [1]), and for zero quark masses and  $G_2 = 0.005 \text{ GeV}^4$  [34], assuming free quarks (with  $V_1$  taken into account in  $L_a$ )

$$\exp \left( -\frac{9V_1(\infty)}{32T_c} \right) T_c = 0.0733 \text{ GeV}, \quad T_c = 0.167 \text{ GeV}. \quad (44)$$

On the other hand, if all quarks above  $T_c$  are bound with gluons, the resulting mass of gluequark can be around  $\frac{9}{4}V_1(\infty, T_c)$  and the corresponding

contribution to the free energy (pressure) is strongly reduced. Neglecting this contribution, one obtains  $T_c$  the same, as in the quenched QCD.

To check accuracy of this prediction, we shall calculate  $T_c$  for the pure gluon case and the case with  $n_f = 3$ ;  $n_a = 0$ . The results are given in Table 3 in comparison to lattice calculation from [35]-[38].

One can see in Table 3 a reasonable agreement with lattice data for the deconfinement transition in case of  $m_q = 0$ .

Now the connection between confinement and CSB in  $SU(N_c)$  theories with fundamental quarks was established in [14], where it was shown, that confinement is a scalar interaction and produces effective scalar quark mass operator.

Moreover, in [15] both  $f_\pi$  and  $\langle \bar{q}q \rangle$  have been calculated in terms of  $\sigma$  in good agreement with lattice and experiment.

It was argued in [14], that the confinement creates in the Green's function of the quark a scalar mass term  $S_q(x, y) = (\hat{\partial} + \hat{\mathcal{M}}(x, y))^{-1}$ ,  $\hat{\mathcal{M}}(x, y) = \sigma |\mathbf{x} - \mathbf{Y}|$ , where  $\bar{Y}$  is an averaged antiquark position (exact in the case of heavy-light mesons). Therefore, CSB occurs automatically in the confinement phase and disappears exactly for fundamental quarks at  $T_{\text{deconf}}$ , when the string tension  $\sigma$  is identically zero, provided that quark mass is small. Note, that for fundamental quarks the bound states with gluons are impossible, however for adjoint quarks there appears another possible source of CSB, namely the possibility of bound  $qg$  system due to nonvanishing of  $D_1(x)$  and hence nonzero  $V_1(r, T)$ , which we can extract from the Polyakov loops  $L_a, L_f$ . To analyze the  $qg$  system in this case one must consider the relativistic hamiltonian (18), but now with replacement of  $\sigma_{\text{adj}} r$  by  $V_1(r, T)$

$$H_{\text{dec}} = 2\sqrt{\mathbf{p}^2} + \frac{9}{4}V_1(r, T) \quad (45)$$

One can estimate approximately  $V_1(\infty, T)$  for different  $T$ , using Polyakov loops measurements of  $L_8, L_3 \equiv L_a, L_f$  in [1], which are done at several values of  $\beta_0 = 1/6g^2$ . Finding the correspondent  $T$  values from the two-loop  $\beta$  function, one obtains that  $V_1(\infty, T)$  from  $L_8, L_3$  in [1] is growing with  $T$  roughly proportionally to  $\sqrt{\sigma_s}$ :  $V_1(\infty, T) \cong 2\sqrt{\sigma_s(t)}$ , where  $\sigma_s(t)$  is the spacial string tension for fundamental quarks. Therefore  $\frac{9}{4}V_1(\infty, T) \approx 1.3$  GeV for  $T \approx 1.8 T_{\text{deconf}}$  and  $\frac{9}{4}V_1(\infty, T) \approx 3$  GeV for  $T \cong 5.3 T_{\text{deconf}}$ . At  $T = T_c$  one has  $L_3 = 0.2$ , which yields  $V_1(\infty, T_c) \cong 0.55$  GeV in good agreement with [28].

Having in mind, that the range of potential  $V_1(r, T)$  is defined by the gluelump mass in  $D_1(x)$  which is of the order of 1 GeV, one can expect, that the interaction in the Hamiltonian (45) is strong enough to produce a bound state of gluequark also for  $T > T_{\text{deconf}}$ . Variational solutions of the equation  $H_{\text{dec}}\Psi_{qg} = m_{qg}\Psi_{qg}$  also support this expectation. A similar situation was observed in the case of white bound states of fundamental quarks or gluons above  $T_{\text{deconf}}$ , see [32], [33], [39]. Leaving details of calculations of AdQCD thermodynamics to the future, we finish here with few remarks on the behavior of the chiral condensate  $\langle\bar{\psi}_a\psi_a\rangle$ , which was measured in [1],[2] in the wide range of temperatures.

At this point one should distinguish two sources of CSB and the chiral condensate  $\langle\bar{\psi}\psi\rangle$ , as was discussed at the end of the previous section. In general, the chiral condensate is nonvanishing both in the limit of small quark mass ( $m_q < m_{\text{crit}}$ ), when spontaneous CSB due to confinement takes place, and also due to nonzero quark mass, which can be dynamically created, e.g. due to  $V_1(r, T)$  as discussed above. It is conceivable, that for large quark mass (whatever is the mass generation mechanism) the quark condensate is also nonzero.

Qualitatively, one can estimate the behavior of the chiral condensate as a function of the large mass of a quark  $m_q$ , as it was done in [34],  $\langle\bar{\psi}\psi\rangle \approx -\frac{G_2}{12m_q}$ . One can expect, that the effective quark mass  $m_q$  is connected to  $m_{qg}$  and is growing with  $T$  as  $V_1(\infty, T)$ , and hence  $\langle\bar{\psi}\psi\rangle$  tends to zero with growing  $T$  due to large  $m_{qg}$ , and due to the fact that bound  $(qg)$  dissociate with growing temperature.

Table 3: Transition temperatures in QCD with fundamental or adjoint quarks from Eq. (39) in comparison with lattice data from [35]-[38].

$N_c$	$n_f$	$n_a$	$T_c(\text{MeV})$ this work	$T_c(\text{MeV})$	$T_c(\text{MeV})$ [37]	$T_c(\text{MeV})$ [38]
3	0	0	260	269 [35]		
3	2	0	180	$173 \pm 8$ [36]		
3	3	0	170	$154 \pm 9$ [36]	164(6)	165(5)(3) 147(2)(3)
3	0	2	167			

Note, that the potential  $V_1(r, T)$  is vector like [40], hence it cannot, in

contrast to  $V_{\text{conf}}$ , produce CSB by itself, and only after it is embedded in the resulting  $(qg)$  mass, which is scalar, this mass breaks chiral symmetry.

Finally few remarks about the so-called PCAC mass  $m$ , defined as in [3, 4] via the ratio of correlators of axial  $A$  and pseudoscalar  $P$  currents.

$$m(t) = \frac{1}{4}[(\partial_0 + \partial_0^*)f_{AP}(t)]/f_{PP}(t). \quad (46)$$

As was discussed in [14], [25],  $m(t)$  can have two regimes, which are seen in the asymptotics in two limiting cases: i)  $t \lesssim \lambda$ ; ii)  $t \gg \lambda$ , where  $\lambda$  is the vacuum correlation length. The latter is defined in FCM [12, 30, 31],  $\lambda \approx 0.1 \div 0.2$  fm. In the regime i)  $m(t) \approx \lambda\sigma$ , while in the case ii)  $m(t \gg \lambda) \approx \sigma\langle r \rangle \approx 2.5\sqrt{\sigma}$ , where  $\langle r \rangle$  is the average size of the meson estimated for linear confinement. This latter large scale regime with  $\sqrt{\sigma}a \approx 0.4ma$  was seemingly observed in the lattice calculations (cf. Fig. 3 of [5]). In the low, scale case  $m(0) = \sigma\lambda (\cong 0.15 \text{ GeV for fundamental quarks})$  defines small resulting values of  $f_\pi, f_K$  as shown in [25]. Note, that all our discussion above in the present paper refers to large scale dynamics only.

## 4 Conclusions

We have studied ADQCD in both confined and deconfined regions, using FCM and applying relativistic hamiltonian [22] to calculate spectrum of bound states. In doing so we have discovered new bound states of quark and gluon  $(qg)$  or gluequarks with masses close to those of glueballs. We have shown, that the appearance of these white fermions drastically changes the whole spectrum and produces in particular large masses of  $PS$  and  $V$  mesons, with the ratio close to unity. These features, and the absolute values of meson masses are in close agreement with lattice data. We have also calculated the deconfinement temperature in AdQCD, which appeared of the same order as in QCD with fundamental quarks, and we have also checked  $T_{\text{deconf}}$  in the latter *vs* lattice data. We have argued, that the  $qg$  bound state may survive above  $T_{\text{deconf}}$  and simulate the nonzero quark condensate, while dissociation of  $qg$  at larger  $T$  may provide restoration of chiral symmetry. The search for gluequarks on the lattice seems highly desirable.

The appearance of massive gluequarks presents itself as a new source of a gauge invariant mass generation mechanism, which can be used at high scale, e.g. in the framework of TC and ETC theories [10, 11].

The author is grateful for useful discussions to A.M.Badalian and M.I.Polokarpov.

## References

- [1] F.Karsch and M.Luetgemeier Nucl. Phys. Proc. Suppl. **73**, 444 (1999); hep-lat/9809056; Nucl. Phys. B **550**, 449 (1999); hep-lat/9812023.
- [2] J.Kogut, J.Polonyi, D.K.Sinclair and H.W.Wyld, Phys. Rev. Lett. **54**,1980 (1985);  
J.Engels, S.Holtmann and T.Schulze, Nucl. Phys. B **724**, 357 (2005);  
G.Cossu, M.D’Elia, A.Di Giacomo, G.Lacagnina and C.Pica, Phys. Rev. D **77**, 074506 (2008).
- [3] L.Del Debbio, B.Lucini, A.Patella, C.Pica and A.Rado, Phys. Rev. D **80**, 074507 (2009); arXiv: 0907.3896 [hep-lat];  
L.Del Debbio, B.Lucini, A.Patella, C.Pica and A.Rado, Phys. Rev. D **82**, 014510 (2010); arXiv: 1004.3206 [hep-lat].
- [4] L.Del Debbio, B.Lucini, A.Patella, C.Pica and A.Rado, Phys. Rev. D **82**, 014509 (2010); arXiv: 1004.3197 [hep-lat].
- [5] A.Patella, L.Del Debbio, B.Lucini, C.Pica and A.Rado, PoS Lattice 2010:068, 2010; arXiv: 1011.0864[hep-lat].
- [6] A.J.Hietanen, J.Rantaharju, K.Rummukainen and K.Tuominen, JHEP **05**, 025 (2009); arXiv:0812.1467 [hep-lat].
- [7] F.Sannino and K.Tuominen, Phys. Rev. D **71**, 051901 (2005).
- [8] D.D.Dietrich and F.Sannino, Phys. Rev. D **75**, 085018 (2007), hep-ph/0611341; S.Catterall and F.Sannini, Phys. Rev. D **76**, 034504 (2007) arXiv: 0705.1664 [hep-lat].
- [9] F.Sannino, arXiv: 0911.0931 [hep-ph].
- [10] C.T.Hill and E.H.Simmons, Phys. Rept. **381**, 235 (2003); hep-ph/0203079.
- [11] K.Lane, arXiv: hep-ph/0202255.
- [12] A.Di Giacomo, H.G.Dosch, V.I.Shevchenko, and Yu.A.Simonov, Phys. Rept., **372**, 319 (2002), hep-ph/0007223; Yu.A.Simonov, in QCD: Perturbative of Nonpertubative, L.S. Fereira, P.Nogueira and J.I.Silva-Marcos eds.; World Scientific, Singapore, 2001; hep-ph/99112337.



- [13] H.G.Dosch, Phys. Lett. B **190**, 177 (1987);  
H.G.Dosch and Yu.A.Simonov, Phys. Lett., B **205**, 339 (1988);  
Yu.A.Simonov, Nucl. Phys. B **307**, 512 (1988); D.S.Kuzmenko,  
V.I.Shevchenko, and Yu.A.Simonov, Phys. Uspekhi **47**, 1 (2004);  
hep-ph/0310190.
- [14] Yu.A.Simonov, Phys. At. Nucl. **60**, 2069 (1997); ibid. **67** 846 (2004);  
Phys. Rev. D **65**, 094018 (2002); hep-ph/0201170.
- [15] Yu.A.Simonov, Phys. At. Nucl. **58**, 309 (1995), hep-ph/9311216;  
Yu.A.Simonov, Ann. Phys. (N.Y.) **323**, 783 (2008); hep-ph/0702266;  
E.V.Komarov and Yu.A.Simonov, Ann. Phys. (N.Y.) **323**, 1230 (2008);  
arXiv:0707.0781 [hep-ph].
- [16] Yu.A.Simonov, Nucl. Phys. B **307**, 512 (1988), Yu.A.Simonov, and  
J.A.Tjon, Ann. Phys. (N.Y.) **300**, 54 (2002); hep-ph/0205165.
- [17] P.de Forcrand and O.Philipsen, Phys. Lett. B **475**, 280 (2009);  
hep-lat/9912950.
- [18] B.S.De Witt, Phys. Rev. **162**, 1195 (1967); ibid. 1239 (1967);  
J.Honerkamp, Nucl. Phys. B **48**, 269 (1972);  
G.'tHooft, Nucl. Phys. B **62**, 44 (1973);  
L.F.Abbot, Nucl. Phys. **185**, 189 (1981);  
Yu.A.Simonov, Phys. At. Nucl. **58**, 107 (1995); Yu.A.Simonov, in: Lec-  
ture Notes in Physics, v. 479, p. 144, Springer, 1996.
- [19] A.B.Kaidalov and Yu.A.Simonov, Phys. Lett. B **477**, 163 (2000);  
hep-ph/9912434; Phys. At. Nucl. **63**, 1428 (2000); hep-ph/9911291;  
Phys. Lett. B **636**, 101 (2006); hep-ph/0512151;  
Yu.A.Simonov, Phys. At. Nucl. **70**, 44 (2007), hep-ph/0603148.
- [20] Yu.A.Simonov, Nucl. Phys. B **592**, 350 (2001); hep-ph/0003114.
- [21] Yu.A.Simonov, JEPT Lett. **57**, 513 (1993); Phys. At. Nucl. **58**, 107  
(1995); hep-ph/9311247.
- [22] A.Yu.Dubin, A.B.Kaidalov and Yu.A.Simonov, Phys. Lett. B **323**, 41  
(1994); Phys. At. Nucl. **56**, 1745 (1993); hep-ph/9311344;  
Yu.S.Kalashnikova, A.V.Nefediev and Yu.A.Simonov, Phys. Rev. D **64**,  
014037 (2001); hep-ph/0103274.

- [23] A.M.Badalian, B.L.G.Bakker, Yu.A.Simonov, Phys. Rev. **D 75**, 116001 (2007); hep-ph/0702157.
- [24] B.Lucini, M.Teper and U.Wenger, JHEP 0406:012, 2004; hep-lat/0404008.
- [25] S.M.Fedorov and Yu.A.Simonov, JETP. Lett. **18**, 57 (2003), hep-ph/0306216; Yu.A.Simonov, Phys. At. Nucl. **67**, 846 (2004); hep-ph/0302090.
- [26] Yu.A.Simonov, Phys. Lett. B **515**, 137 (2001); hep-ph/0105141; A.Di Giacomo and Yu.A.Simonov, Phys. Lett. B **595**, 368 (2004); hep-ph/0404044.
- [27] Yu.A.Simonov, JETP Lett. **54**, 249 (1991); *ibid*, **55**, 627 (1992); Phys. Atom. Nucl. **58**, 309 (1995); hep-ph/9311216.  
H. G. Dosch, H.J.Pirner and Yu. A. Simonov, Phys. Lett. B. **349**, 335 (1995).
- [28] Yu.A.Simonov and M.A.Trusov, Phys. Lett. B **650**, 36 (2007); arXiv: 0703277 [hep-ph].
- [29] A.V.Nefediev, Yu.A.Simonov, M.A.Trusov, Int. J. Mod. Phys. **E 18**, 549 (2009).
- [30] M.D’Elia, A.Di Giacomo, and E.Meggiolaro, Phys. Lett B **408**, 315 (1997); Phys. Rev. D **67**, 114504 (2003);  
A.Di Giacomo, E.Meggiolaro and H.Panagopoulos, Nucl. Phys. B **483**, 371 (1997).
- [31] V.I.Shevchenko and Yu.A.Simonov, Adv. HEPH, 87305 (2009); arXiv: 0902.1405 [hep-ph].  
Yu.A.Simonov, Trudy Mat. Inst. im. V.A.Steklova **272**, 2344 (2010), arXiv: 1003.3608 [hep-ph].
- [32] Yu.A.Simonov, Phys. Lett. B **619**, 293 (2005); hep-ph/0502078.
- [33] A.Di Giacomo, E.Meggiolaro, Yu.A.Simonov and A.I.Veselov, Phys. At. Nucl. **70**, 908 (2007), hep-ph/0512125.

- [34] M.A.Shifman, A.I.Vainshtein, and V.I.Zakharov, Nucl. Phys. **B 147**, 385 (1979);  
B.L.Ioffe, Prog. Part. Nucl. Phys. **56**, 232 (2006); hep-ph/0502148.
- [35] O.Kaczmarek, F.Karsch, P.Petreczky and F.Zantov, Phys. Lett. B **543**, 41 (2002); hep-lat/0207002.
- [36] F.Karsch, Nucl. Phys. B (Proc. Suppl) **83**, 14 (2000); F.Karsch, E.Laermann and A.Peikert, hep-lat/0012023.
- [37] A.Bazavov, T.Bhattacharya, M. Cheng, C.DeTar, et al., arXiv: 1111.1710 [hep-lat].
- [38] S.Borsanyi, Z.Fodor, C.Hoelbling, S.D.Katz et al., JHEP 1009.073 (2009); arXiv: 1005. 3508 [hep-lat].
- [39] P.Petreczky, arXiv: 1001.5284 [hep-ph].
- [40] A.V.Nefediev and Yu.A.Simonov, Phys. Rev. D **76** , 074014, (2007); arXiv: 0708.3603 [hep-ph].